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CONSTRUCTION OF STRONG RATIONAL DIOPHANTINE TRIPLE WITH PROPERTY D (K2) USING THE SOLUTIONS OF SPECIAL THREE DIMENSIONAL SURFACES

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ABSTRACT

we search for three distinct polynomials with integer co-efficients such that the sum of any two added with either an arbitrary integer or a polynomial with integer co-efficients is a perfect square of a polynomial with integer co-efficients

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I. INTRODUCTION

The problem of constructing the set with property that the product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus[1]. A set of m non-zero distinct positive integers $\{a_1, a_2 ......a_m\}$ is called Diophantine if $a_i a_j + 1 =$, a perfect square and such a set is said to be a Diophantine m-tuples with property D(1). Diophantus found the first Diophantine quadruple of rational numbers $\{\frac{1}{16}, \frac{33}{16}, \frac{17}{4}, \frac{105}{16}\}$ [1] while the first set of four positive integers with the above property was found by Fermat and it was $\{1,3,8,120\}$. Euler gave the solution $\{a, b, a+b+2r, 4r(r+a)(r+b)\}$ where $a = b+1 = r^2$ [2]. Many mathematicians considered the problem of the existence of Diophantine quadruples with the property D(n) for any arbitrary integer n [3-9] and also for any non-zero polynomials in n with integer coefficients[10-16] Many generalizations of this problem were considered since antiquity for example, by adding a fixed integer n instead of 1, looking at k^{th} powers instead of squares or considering the problems over domains other than Z or Q. For an extensive review of various articles on Diophantine m-tuples, one may refer the website http:// web.math.pmf.unizg.hr/~duje/ref.html. These results motivated us for determining polynomial triples with integer coefficients such that the sum of any two added with either an arbitrary integer or a polynomial with integer coefficients is a perfect square of a polynomial with integer coefficients

II. SECTION:1 DIOPHANTINE TRIPLE USING PYTHAGOREAN SOLUTION

Let $a = \frac{-kx}{z}$ and $a = \frac{kx}{z}$ be two rational numbers where x and z represent a leg and hypotenuse of the Pythagorean triangle $T(x,y,z)$

Now $$ab + k^2 = (\frac{ky}{z})^2 = r_1^2 \text{ (say)}$$

Therefore the pair (a,b) is a rational Diophantine two-tuple with property $D(k^2)$

Let c be any non-zero rational number such that

$$ac + D(k^2) = \alpha^2$$

$$bc + D(k^2) = \beta^2$$

from which we get

$$c = \frac{2ky}{z}$$

Thus, the triple (a, b, c) is a rational Diophantine three- tuple with property $D(k^2)$

Since

$$ac + k^2 = \left(\frac{k(x-y)}{z}\right)^2$$

note that the above triple (a,b,c) is a strong rational Diophantine triple as the product of any two members of the triple added with k^2 is a perfect square.

Now, consider the pair (b, c) which is a rational Diophantine two tuple with property $D(k^2)$. Applying Euler's formula, it is seen that the triple (b, c, d) is a strong rational Diophantine three tuple with property$D(k^2)$,where $d = \frac{k(3x+4y)}{z}$. The repetition of the above process leads to the generation of sequence of strong rational Diophantine triples with property$D(k^2)$

A few numerical examples are presented in the table:

x	y	z	k	(a,b,c)	(b,c,d)	(c,d,e)	(d,e,f)
4	3	5	1	$(\frac{-4}{5},\frac{4}{5},\frac{6}{5})$	$\left(\frac{4}{5},\frac{6}{5},\frac{24}{5}\right)$	$\left(\frac{6}{5},\frac{24}{5},\frac{56}{5}\right)$	$\left(\frac{24}{5},\frac{56}{5},\frac{154}{5}\right)$
3	4	5	2	$(\frac{6}{5},\frac{6}{5},\frac{16}{5})$	$\left(\frac{6}{5},\frac{16}{5},10\right)$	$\left(\frac{16}{5},10,\frac{126}{5}\right)$	$\left(10,\frac{126}{5},\frac{336}{5}\right)$
12	5	13	2	$(\frac{-24}{13},\frac{24}{13},\frac{20}{13})$	$\left(\frac{24}{13},\frac{20}{13},\frac{112}{13}\right)$	$\left(\frac{20}{13},\frac{112}{13},\frac{240}{13}\right)$	$\left(\frac{112}{13},\frac{240}{13},\frac{684}{13}\right)$
5	12	13	3	$\left(\frac{-15}{13},\frac{15}{13},\frac{72}{13}\right)$	$\left(\frac{15}{13},\frac{72}{13},\frac{189}{13}\right)$	$\left(\frac{72}{13},\frac{189}{13},\frac{507}{13}\right)$	$\left(\frac{189}{13},\frac{507}{13},\frac{1320}{13}\right)$

III. SECTION:2 DIOPHANTINE TRIPLE USING SOLUTIONS OF ELLIPTIC PARABOLOID

Let $a = \frac{-2kx^2}{z}$, $b = \frac{2ky^2}{z}$ be two rational numbers where (x, y, z) satisfies the elliptic paraboloid $x^2 + y^2$ =z

Now, $$ab + k^2 = [\left(\frac{y^2-x^2}{x^2+y^2}\right)k]^2 = r_1^2 \quad \text{(say)}$$

Therefore, the pair (a, b) is a rational Diophantine two-tuple with property $D(k^2)$

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Following the analysis similar to section:1, the corresponding strong rational Diophantine triples

(a,b,c), (a,c,d), (a,d,e), ……………..with property $D(k^2)$ are given by

$$\left(\frac{-2kx^2}{z},\frac{2ky^2}{z},\frac{4k(y^2-x^2)}{z}\right),\left(\frac{-2kx^2}{z},\frac{4k(y^2-x^2)}{z},\frac{6k(y^2-2x^2)}{z}\right),$$

$\left(\frac{-2kx^2}{z},\frac{6k(y^2-2x^2)}{z},\frac{8k(y^2-3x^2)}{z}\right)$………………

A few numerical examples are presented below:

x	y	z	k	(a,b,c)	(a,c,d)	(a,d,e)	(a,e,f)
2	3	13	1	$\left(\frac{-8}{13},\frac{18}{13},\frac{20}{13}\right)$	$\left(\frac{-8}{13},\frac{20}{13},\frac{6}{13}\right)$	$\left(\frac{-8}{13},\frac{6}{13},\frac{-24}{13}\right)$	$\left(\frac{-8}{13},\frac{-24}{13},\frac{-70}{13}\right)$
3	5	34	2	$\left(\frac{-36}{34},\frac{100}{34},\frac{128}{34}\right)$	$\left(\frac{-36}{34},\frac{128}{34},\frac{84}{34}\right)$	$\left(\frac{-36}{34},\frac{84}{34},\frac{-32}{34}\right)$	$\left(\frac{-36}{34},\frac{-32}{34},\frac{-220}{34}\right)$
4	6	52	1	$\left(\frac{-32}{52},\frac{72}{52},\frac{80}{52}\right)$	$\left(\frac{-32}{52},\frac{80}{52},\frac{24}{52}\right)$	$\left(\frac{-32}{52},\frac{24}{52},\frac{96}{52}\right)$	$\left(\frac{-32}{52},\frac{96}{52},\frac{-280}{52}\right)$
1	4	17	2	$\left(\frac{-4}{17},\frac{64}{17},\frac{120}{17}\right)$	$\left(\frac{-4}{17},\frac{120}{17},\frac{168}{17}\right)$	$\left(\frac{-4}{17},\frac{168}{17},\frac{208}{17}\right)$	$\left(\frac{-4}{17},\frac{208}{17},\frac{240}{17}\right)$

NOTE: It is worth to note that the above sequence of triples may be represented in general form as the triple $\left(\frac{-2kx^2}{z},\frac{4\alpha k(y^2-(\alpha-1)x^2)}{z},\frac{(2\alpha+2)k(y^2-\alpha x^2)}{z}\right)$ where $\alpha = 1,2,3$ … … … … ..

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IV. CONCLUSION

In this paper, polynomial triples with special numbers as members are constructed such that the sum of any two members of the triple added with either an integer or a polynomial is a perfect square of polynomial with integer coefficients. Since numbers are rich in variety, one may search for polynomial triples with higher order number patterns.

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